Vibrations of rotors partially filled with liquids in hydrodynamically lubricated journal bearings

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16th February 2017

TECHNISCHE UNIVERSITÄT
CHEMNITZ

Motivation

http://www.miele.de

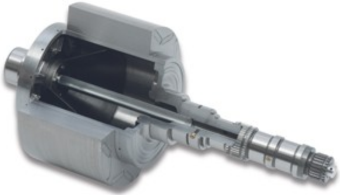
washing machines

Motivation

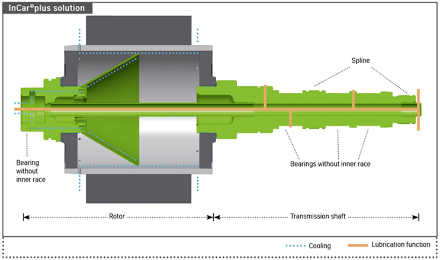
http://www.lanner.de

centrifuges

Motivation



http://incarplus.thyssenkrupp.com



functional integration (lubrication, cooling) in electrical drives

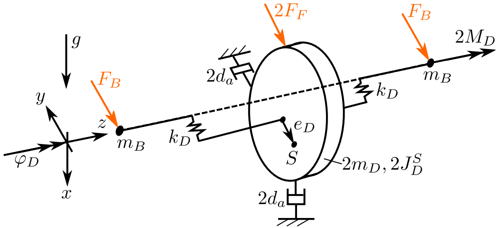
Literature

- General rotordynamics
  - Laval 1883
  - Gasch & Nordmann & Pfützner 2006
- Rotors in hydrodynamically lubricated journal bearings
  - Reynolds 1886
  - Sommerfeld 1955
  - Moser 1993
- Fluid filled rigid bodies
  - Stokes 1847
  - Kollmann 1962
  - Moiseyev & Rumyantsev 1968
  - Ibrahim 2005
  - Derendyaev & Vostrukhov & Soldatov 2006

Outline

- Modelling
  - rotor model
  - bearing model
  - liquid filling model
- Results
  - transient run-up simulation
  - bifurcation analysis of stationary solutions

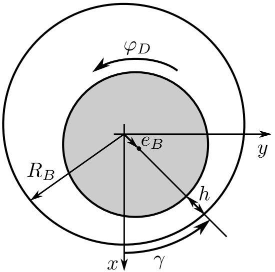
Rotor model



Laval-like rotor

$$
\begin{aligned}
m_B\ddot{x}_B + k_D(x_B - x_D) &= F_{Bx} + m_B g \\
m_B\ddot{y}_B + k_D(y_B - y_D) &= F_{By} \\
m_D\ddot{x}_D + d_a\dot{x}_D + k_D(x_D - x_B) &= F_{Fx} + m_D e_D(\dot{\varphi}_D^2 \cos\varphi_D + \ddot{\varphi}_D \sin\varphi_D) \\
&\quad + m_D g \\
m_D\ddot{y}_D + d_a\dot{y}_D + k_D(y_D - y_B) &= F_{Fy} + m_D e_D(\dot{\varphi}_D^2 \sin\varphi_D - \ddot{\varphi}_D \cos\varphi_D) \\
J_D^S\ddot{\varphi}_D &= M_F + e_D(k_D y_D + d_a\dot{y}_D)\cos\varphi_D \\
&\quad M_D - e_D(k_D x_D + d_a\dot{x}_D)\sin\varphi_D
\end{aligned}
$$

Bearing model

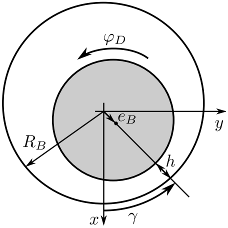


Rotor shaft in radial bearings

Reynolds' equation describes the pressure distribution in the lubrication film

$$\frac{1}{R_B^2}\frac{\partial}{\partial\varphi}\left(\frac{h^3}{\eta_B}\frac{\partial p}{\partial z}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\eta_B}\frac{\partial p}{\partial z}\right)=12\frac{\partial h}{\partial t}+6\omega\frac{\partial h}{\partial\varphi}$$

Bearing model



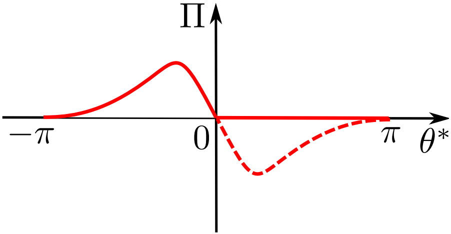
Rotor shaft in radial bearings

nondimensionalization reveals simplification for short bearings

$$\bar{z} = \frac{2z}{B_B}, \qquad H = \frac{h}{C}, \qquad \tau = \omega t, \qquad \Pi = \frac{C^2}{R_B^2}\frac{p}{\eta_B \omega},$$

$$\frac{\partial}{\partial \varphi}\left(H^3 \frac{\partial \Pi}{\partial \bar{z}}\right) + \underbrace{\left(\frac{2R_B}{B_B}\right)^2}_{\gg 1} \frac{\partial}{\partial \bar{z}}\left(H^3 \frac{\partial \Pi}{\partial \bar{z}}\right) = 12\frac{\partial H}{\partial \tau} + 6\frac{\partial H}{\partial \varphi}$$

Bearing model

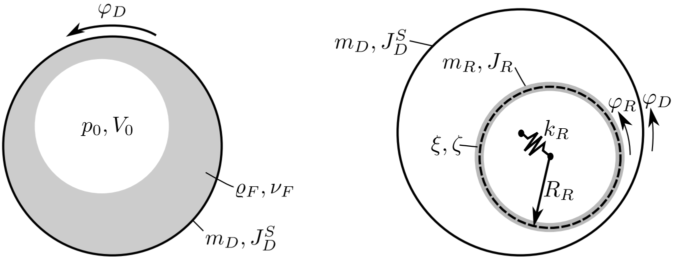


Gümbel boundary condition

$$F_{Bx}(e_B, e'_B, \gamma, \gamma') = -R_B \int_{-B_B/2}^{B_B/2} \int_{\Phi_1}^{\Phi_2} p(\Phi, z, e_B, e'_B, \gamma, \gamma') \cos\Phi \, \mathrm{d}\Phi \mathrm{d}z,$$

$$F_{By}(e_B, e'_B, \gamma, \gamma') = -R_B \int_{-B_B/2}^{B_B/2} \int_{\Phi_1}^{\Phi_2} p(\Phi, z, e_B, e'_B, \gamma, \gamma') \sin\Phi \, \mathrm{d}\Phi \mathrm{d}z.$$

Liquid filling



Liquid model and its reduction to a rigid body

friction force between rotor (“disk”) and liquid (“ring”)

$$\mathbf{f}_\eta = -\xi(\mathbf{v}_R - \mathbf{v}_D) - \zeta \mathbf{e}_z \times (\mathbf{v}_R - \mathbf{v}_D).$$

integrated along contact line of length $L = 2\pi R_R$ contributes to resulting force

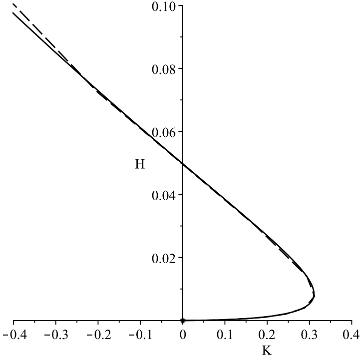
$$\begin{aligned}
F_{Fx} &= -k_R(x_R - x_D) - \xi L\big((\dot{x}_R - \dot{x}_D) + (y_R - y_D)\dot{\varphi}_D\big) \\
&\quad + \zeta L\big((\dot{y}_R - \dot{y}_D) - (x_R - x_D)\dot{\varphi}_D\big) \\
F_{Fy} &= -k_R(y_R - y_D) - \xi L\big((\dot{y}_R - \dot{y}_D) - (x_R - x_D)\dot{\varphi}_D\big) \\
&\quad - \zeta L\big((\dot{x}_R - \dot{x}_D) + (y_R - y_D)\dot{\varphi}_D\big)
\end{aligned}$$

continuous (m_F, ν_F, δ) $\Rightarrow$ discrete $(m_R, k_R, \xi L, \zeta L)$

$$K_c = \frac{k_B}{m_F \omega_D^2} \qquad K_d = \frac{k_B}{m_R \omega_D^2}$$

$$H_c = \frac{d_B}{m_F \omega_D} \qquad H_d = \frac{d_B}{m_R \omega_D^2}$$

$$\omega_P / \omega_D = 0 \ldots 0.7$$



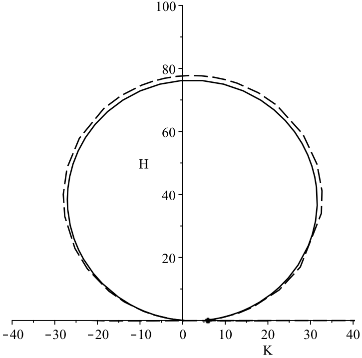
stability limit for a balanced, rigid rotor partially filled with liquid in isotropic, linear visco-elastic bearings and without external force fields

continuous (m_F, ν_F, δ) $\Rightarrow$ discrete $(m_R, k_R, \xi L, \zeta L)$

$$K_c = \frac{k_B}{m_F \omega_D^2} \qquad K_d = \frac{k_B}{m_R \omega_D^2}$$

$$H_c = \frac{d_B}{m_F \omega_D} \qquad H_d = \frac{d_B}{m_R \omega_D^2}$$

$$\omega_P / \omega_D = 0 \ldots 1$$



stability limit for a balanced, rigid rotor partially filled with liquid in isotropic, linear visco-elastic bearings and without external force fields

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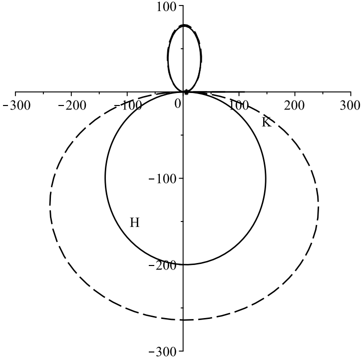
$$K_c = \frac{k_B}{m_F \omega_D^2} \qquad K_d = \frac{k_B}{m_R \omega_D^2}$$

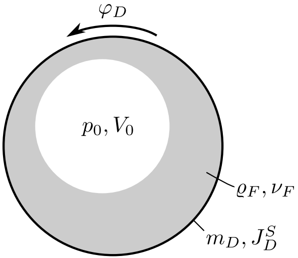
$$H_c = \frac{d_B}{m_F \omega_D} \qquad H_d = \frac{d_B}{m_R \omega_D^2}$$

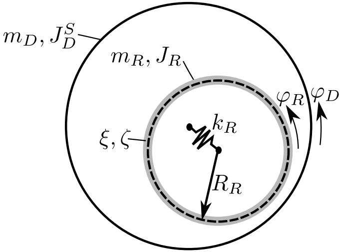
$$\omega_P / \omega_D = 0 \ldots 10$$



stability limit for a balanced, rigid rotor partially filled with liquid in isotropic, linear visco-elastic bearings and without external force fields

Limitations of the reduced model





the discrete model approximates the continuous model under the assumptions of

- **a ring-shaped distribution of the liquid in the rotor**, i.e. the rotational speed must be sufficiently high and be reached without prior instabilities
- **only the slow wave mode is approximated accurately**, i.e. the accelerations must be slowly enough not to excite higher wave modes

Physical model

for prescribed rotational speed $\dot{\varphi}_D(t)$ translational and rotational motion decouple

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{x}} + (\mathbf{D} + \mathbf{G})\dot{\mathbf{x}} + (\mathbf{K} + \mathbf{Z})\mathbf{x} &= \mathbf{f}(t, \mathbf{x}, \dot{\mathbf{x}}) \\ J_R\ddot{\varphi}_R + \xi L R_R^2 \dot{\varphi}_R &= \xi L R_R^2 \dot{\varphi}_D(t) \end{aligned}$$

$$\mathbf{M} = \begin{bmatrix} m_B & 0 & 0 & 0 & 0 & 0 \\ 0 & m_B & 0 & 0 & 0 & 0 \\ 0 & 0 & m_D & 0 & 0 & 0 \\ 0 & 0 & 0 & m_D & 0 & 0 \\ 0 & 0 & 0 & 0 & m_R & 0 \\ 0 & 0 & 0 & 0 & 0 & m_R \end{bmatrix}$$

Physical model

for prescribed rotational speed $\dot{\varphi}_D(t)$ translational and rotational motion decouple

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{x}} + (\mathbf{D}+\mathbf{G})\dot{\mathbf{x}} + (\mathbf{K}+\mathbf{Z})\mathbf{x} &= \mathbf{f}(t,\mathbf{x},\dot{\mathbf{x}}) \\ J_R\ddot{\varphi}_R + \xi L R_R^2 \dot{\varphi}_R &= \xi L R_R^2 \dot{\varphi}_D(t) \end{aligned}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_a+\xi L & 0 & -\xi L & 0 \\ 0 & 0 & 0 & d_a+\xi L & 0 & -\xi L \\ 0 & 0 & -\xi L & 0 & \xi L & 0 \\ 0 & 0 & 0 & -\xi L & 0 & \xi L \end{bmatrix}$$

Physical model

for prescribed rotational speed $\dot{\varphi}_D(t)$ translational and rotational motion decouple

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{x}} + (\mathbf{D}+\mathbf{G})\dot{\mathbf{x}} + (\mathbf{K}+\mathbf{Z})\mathbf{x} &= \mathbf{f}(t,\mathbf{x},\dot{\mathbf{x}}) \\ J_R\ddot{\varphi}_R + \xi L R_R^2 \dot{\varphi}_R &= \xi L R_R^2 \dot{\varphi}_D(t) \end{aligned}$$

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\zeta L & 0 & \zeta L \\ 0 & 0 & \zeta L & 0 & -\zeta L & 0 \\ 0 & 0 & 0 & \zeta L & 0 & -\zeta L \\ 0 & 0 & -\zeta L & 0 & \zeta L & 0 \end{bmatrix}$$

Physical model

for prescribed rotational speed $\dot{\varphi}_D(t)$ translational and rotational motion decouple

$$
\begin{aligned}
\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{D}+\mathbf{G})\dot{\mathbf{x}} + (\mathbf{K}+\mathbf{Z})\mathbf{x} &= \mathbf{f}(t,\mathbf{x},\dot{\mathbf{x}}) \\
J_R\ddot{\varphi}_R + \xi L R_R^2 \dot{\varphi}_R &= \xi L R_R^2 \dot{\varphi}_D(t)
\end{aligned}
$$

$$
\mathbf{K} = \begin{bmatrix}
k_D & 0 & -k_D & 0 & 0 & 0 \\
0 & k_D & 0 & -k_D & 0 & 0 \\
-k_D & 0 & k_D + \tilde{k} & 0 & -\tilde{k} & 0 \\
0 & -k_D & 0 & k_D + \tilde{k} & 0 & -\tilde{k} \\
0 & 0 & -\tilde{k} & 0 & \tilde{k} & 0 \\
0 & 0 & 0 & -\tilde{k} & 0 & \tilde{k}
\end{bmatrix}
$$

with $\tilde{k} = k_R + \zeta L \dot{\varphi}_D$

Physical model

for prescribed rotational speed $\dot{\varphi}_D(t)$ translational and rotational motion decouple

$$
\begin{aligned}
\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{D}+\mathbf{G})\dot{\mathbf{x}} + (\mathbf{K}+\mathbf{Z})\mathbf{x} &= \mathbf{f}(t,\mathbf{x},\dot{\mathbf{x}}) \\
J_R\ddot{\varphi}_R + \xi L R_R^2 \dot{\varphi}_R &= \xi L R_R^2 \dot{\varphi}_D(t)
\end{aligned}
$$

$$
\mathbf{Z} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \xi L\dot{\varphi}_D & 0 & -\xi L\dot{\varphi}_D \\
0 & 0 & -\xi L\dot{\varphi}_D & 0 & \xi L\dot{\varphi}_D & 0 \\
0 & 0 & 0 & -\xi L\dot{\varphi}_D & 0 & \xi L\dot{\varphi}_D \\
0 & 0 & \xi L\dot{\varphi}_D & 0 & -\xi L\dot{\varphi}_D & 0
\end{bmatrix}
$$

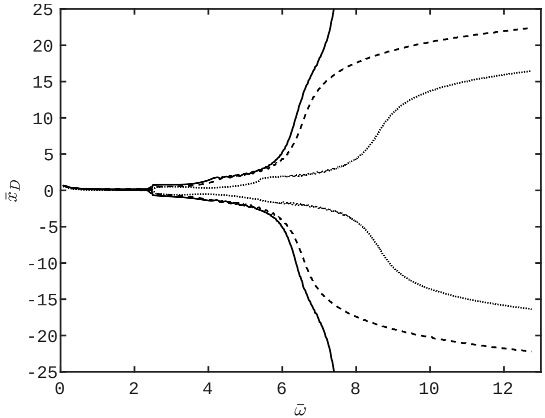
Physical model

for prescribed rotational speed $\dot{\varphi}_D(t)$ translational and rotational motion decouple

$$\begin{aligned}\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{D}+\mathbf{G})\dot{\mathbf{x}} + (\mathbf{K}+\mathbf{Z})\mathbf{x} &= \mathbf{f}(t,\mathbf{x},\dot{\mathbf{x}}) \\ J_R\ddot{\varphi}_R + \xi L R_R^2 \dot{\varphi}_R &= \xi L R_R^2 \dot{\varphi}_D(t)\end{aligned}$$

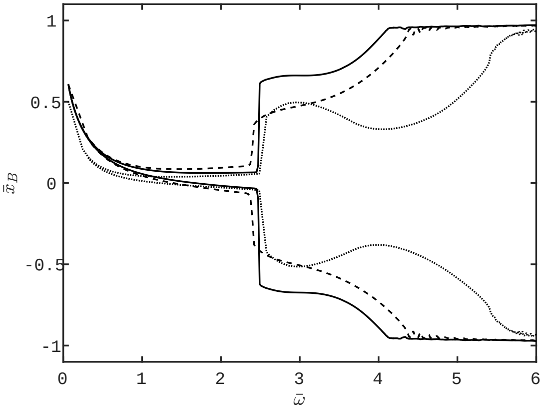
$$\mathbf{f} = \begin{bmatrix} F_{Bx}(x_B,\dot{x}_B,y_B,\dot{y}_B) + m_B g \\ F_{By}(x_B,\dot{x}_B,y_B,\dot{y}_B) \\ m_D e_D(\dot{\varphi}_D^2 \cos\varphi_D + \ddot{\varphi}_D \sin\varphi_D) + m_D g \\ m_D e_D(\dot{\varphi}_D^2 \sin\varphi_D - \ddot{\varphi}_D \cos\varphi_D) \\ m_R g \\ 0 \end{bmatrix}$$

Run-up simulation $\dot{\varphi}_D(t) = \omega_s + \alpha t$



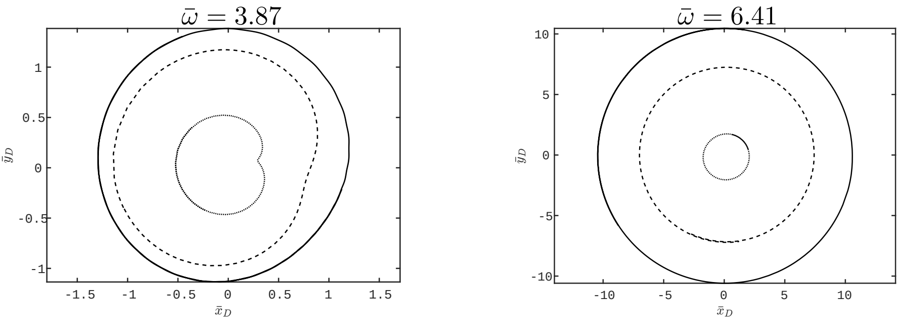
displacement of rotor disk for empty rotor (dotted line), filled rotor (solid line) and equivalent-mass rotor (dashed line)

Run-up simulation $\dot{\varphi}_D(t) = \omega_s + \alpha t$



displacement of rotor shaft in bearings for empty rotor (dotted line), filled rotor (solid line) and equivalent-mass rotor (dashed line)

Run-up simulation $\dot{\varphi}_D(t) = \omega_s + \alpha t$



trajectories of the rotor disk for empty rotor (dotted line), filled rotor (solid line) and equivalent-mass rotor (dashed line)

Nondimensionalization

coordinates	$\bar{x}_i = \frac{x_i}{C}, \quad \bar{y}_i = \frac{y_i}{C}$	$i = B, D, (R)$
time	$\tau = \omega t$	$\omega = \dot{\varphi}_D =$ constant
angular frequency	$\bar{\omega} = \omega\sqrt{C/g}$	
masses	$\bar{m}_i = \frac{m_i}{m}$	$i = B, D, (R)$
damping/friction	$\bar{d}_i = \frac{d_i}{m}\sqrt{\frac{C}{g}}$	$i = a, \xi, \zeta$
stiffnesses	$\bar{k}_i = \frac{C}{mg}k_i$	$i = D, (R)$
imbalance	$\rho = \frac{e_D}{C}$	
reciprocal load parameter	$\sigma = \frac{1}{2}\frac{R_B B_B^3 \eta_B}{C^2 m\sqrt{Cg}}$	$S_m = \sigma\bar{\omega}$
moment of inertia	$\bar{J}_R = \frac{J_R}{mR_R C}$	
rotational damping	$\bar{d}_R = \frac{\xi L R_R}{m\sqrt{Cg}}$	

Dimensionless model

$$\begin{aligned}
\bar{\mathbf{M}}\bar{\omega}^2\bar{\mathbf{x}}'' + (\bar{\mathbf{D}} + \bar{\mathbf{G}})\bar{\omega}\bar{\mathbf{x}}' + (\bar{\mathbf{K}}_0 + \bar{\mathbf{K}}_1\bar{\omega})\bar{\mathbf{x}} &= \bar{\mathbf{f}}(\tau, \bar{\mathbf{x}}, \bar{\mathbf{x}}') \\
\bar{J}_R\bar{\omega}^2\bar{\varphi}_R'' + \bar{d}_R\bar{\omega}\bar{\varphi}_R' &= \bar{d}_R\bar{\omega}
\end{aligned}$$

$$\mathbf{M} = \begin{bmatrix} \bar{m}_B & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{m}_B & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{m}_D & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{m}_D & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{m}_R & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{m}_R \end{bmatrix}$$

Dimensionless model

$$\begin{aligned}
\bar{\mathbf{M}}\bar{\omega}^2\bar{\mathbf{x}}'' + (\bar{\mathbf{D}} + \bar{\mathbf{G}})\bar{\omega}\bar{\mathbf{x}}' + (\bar{\mathbf{K}}_0 + \bar{\mathbf{K}}_1\bar{\omega})\bar{\mathbf{x}} &= \bar{\mathbf{f}}(\tau, \bar{\mathbf{x}}, \bar{\mathbf{x}}') \\
\bar{J}_R\bar{\omega}^2\bar{\varphi}_R'' + \bar{d}_R\bar{\omega}\bar{\varphi}_R' &= \bar{d}_R\bar{\omega}
\end{aligned}$$

$$\bar{\mathbf{D}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \bar{d}_a + \bar{d}_\xi & 0 & -\bar{d}_\xi & 0 \\
0 & 0 & 0 & \bar{d}_a + \bar{d}_\xi & 0 & -\bar{d}_\xi \\
0 & 0 & -\bar{d}_\xi & 0 & \bar{d}_\xi & 0 \\
0 & 0 & 0 & -\bar{d}_\xi & 0 & \bar{d}_\xi
\end{bmatrix}$$

Dimensionless model

$$
\begin{aligned}
\bar{\mathbf{M}}\bar{\omega}^2\bar{\mathbf{x}}'' + (\bar{\mathbf{D}} + \bar{\mathbf{G}})\bar{\omega}\bar{\mathbf{x}}' + (\bar{\mathbf{K}}_0 + \bar{\mathbf{K}}_1\bar{\omega})\bar{\mathbf{x}} &= \bar{\mathbf{f}}(\tau, \bar{\mathbf{x}}, \bar{\mathbf{x}}') \\
\bar{J}_R\bar{\omega}^2\bar{\varphi}_R'' + \bar{d}_R\bar{\omega}\bar{\varphi}_R' &= \bar{d}_R\bar{\omega}
\end{aligned}
$$

$$
\bar{\mathbf{G}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\bar{d}_\zeta & 0 & \bar{d}_\zeta \\
0 & 0 & \bar{d}_\zeta & 0 & -\bar{d}_\zeta & 0 \\
0 & 0 & 0 & \bar{d}_\zeta & 0 & -\bar{d}_\zeta \\
0 & 0 & -\bar{d}_\zeta & 0 & \bar{d}_\zeta & 0
\end{bmatrix}
$$

Dimensionless model

$$
\begin{aligned}
\bar{\mathbf{M}}\bar{\omega}^2\bar{\mathbf{x}}'' + (\bar{\mathbf{D}} + \bar{\mathbf{G}})\bar{\omega}\bar{\mathbf{x}}' + (\bar{\mathbf{K}}_0 + \bar{\mathbf{K}}_1\bar{\omega})\bar{\mathbf{x}} &= \bar{\mathbf{f}}(\tau, \bar{\mathbf{x}}, \bar{\mathbf{x}}') \\
\bar{J}_R\bar{\omega}^2\bar{\varphi}_R'' + \bar{d}_R\bar{\omega}\bar{\varphi}_R' &= \bar{d}_R\bar{\omega}
\end{aligned}
$$

$$
\bar{\mathbf{K}}_0 = \begin{bmatrix}
\bar{k}_D & 0 & -\bar{k}_D & 0 & 0 & 0 \\
0 & \bar{k}_D & 0 & -\bar{k}_D & 0 & 0 \\
-\bar{k}_D & 0 & \bar{k}_D + \bar{k}_R & 0 & -\bar{k}_R & 0 \\
0 & -\bar{k}_D & 0 & \bar{k}_D + \bar{k}_R & 0 & -\bar{k}_R \\
0 & 0 & -\bar{k}_R & 0 & \bar{k}_R & 0 \\
0 & 0 & 0 & -\bar{k}_R & 0 & \bar{k}_R
\end{bmatrix}
$$

Dimensionless model

$$
\begin{aligned}
\bar{\mathbf{M}}\bar{\omega}^2\bar{\mathbf{x}}'' + (\bar{\mathbf{D}} + \bar{\mathbf{G}})\bar{\omega}\bar{\mathbf{x}}' + (\bar{\mathbf{K}}_0 + \bar{\mathbf{K}}_1\bar{\omega})\bar{\mathbf{x}} &= \bar{\mathbf{f}}(\tau, \bar{\mathbf{x}}, \bar{\mathbf{x}}') \\
\bar{J}_R\bar{\omega}^2\bar{\varphi}_R'' + \bar{d}_R\bar{\omega}\bar{\varphi}_R' &= \bar{d}_R\bar{\omega}
\end{aligned}
$$

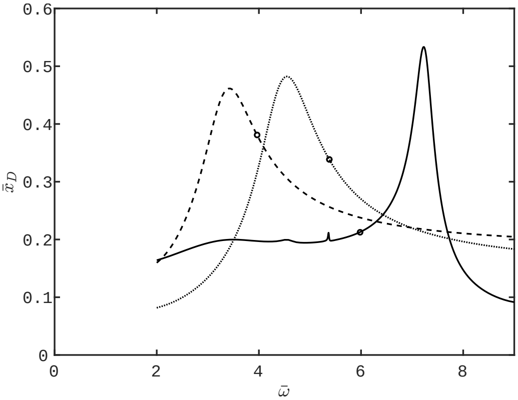
$$
\bar{\mathbf{K}}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{d}_\zeta & 0 & -\bar{d}_\zeta & 0 \\ 0 & 0 & 0 & \bar{d}_\zeta & 0 & -\bar{d}_\zeta \\ 0 & 0 & -\bar{d}_\zeta & 0 & \bar{d}_\zeta & 0 \\ 0 & 0 & 0 & -\bar{d}_\zeta & 0 & \bar{d}_\zeta \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{d}_\xi & 0 & -\bar{d}_\xi \\ 0 & 0 & -\bar{d}_\xi & 0 & \bar{d}_\xi & 0 \\ 0 & 0 & 0 & -\bar{d}_\xi & 0 & \bar{d}_\xi \\ 0 & 0 & \bar{d}_\xi & 0 & -\bar{d}_\xi & 0 \end{bmatrix}
$$

Dimensionless model

$$
\begin{aligned}
\bar{\mathbf{M}}\bar{\omega}^2\bar{\mathbf{x}}'' + (\bar{\mathbf{D}} + \bar{\mathbf{G}})\bar{\omega}\bar{\mathbf{x}}' + (\bar{\mathbf{K}}_0 + \bar{\mathbf{K}}_1\bar{\omega})\bar{\mathbf{x}} &= \bar{\mathbf{f}}(\tau, \bar{\mathbf{x}}, \bar{\mathbf{x}}') \\
\bar{J}_R\bar{\omega}^2\bar{\varphi}_R'' + \bar{d}_R\bar{\omega}\bar{\varphi}_R' &= \bar{d}_R\bar{\omega}
\end{aligned}
$$

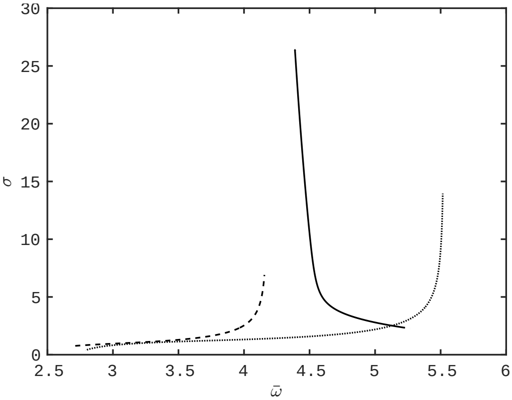
$$
\mathbf{f} = \begin{bmatrix}
S_m f_x(\bar{x}_B, \bar{x}_B', \bar{y}_B, \bar{y}_B') + \bar{m}_B \\
S_m f_y(\bar{x}_B, \bar{x}_B', \bar{y}_B, \bar{y}_B') \\
\bar{m}_D \rho \bar{\omega}^2 \cos\tau + \bar{m}_D \\
\bar{m}_D \rho \bar{\omega}^2 \sin\tau \\
\bar{m}_R \\
0
\end{bmatrix}
$$

Bifurcation analysis



dimensionless rotor angular velocity versus dimensionless rotor disk position for empty rotor (dotted line), filled rotor (solid line) and equivalent-mass rotor (dashed line)

Bifurcation analysis



path of the first bifurcation point in dependence on dimensionless angular velocity and dimensionless reciprocal load parameter for empty rotor (dotted line), filled rotor (solid line) and equivalent-mass rotor (dashed line)

Summary

- liquid filled rotors in hydrodynamically lubricated journal bearings have been reduced to a minimal model with 6-DoF (prescribed rotational speed) which is well suited for repeated evaluations and inclusion in further studies
- the liquid filling has a major influence on the rotor dynamics, so far only destabilizing effects have been observed

Outlook

- verify the results by comparison with reference results from the literature
- develop model order reduction of the liquid ("continuous model") to the rigid body ("discrete model") into functional expressions
- investigate influence of all parameters (11 groups)
- search for synchronization and stabilization effects

parameter values

R_B	3e-3 m
B_B	3e-3 m
C	1e-5 m
η_B	10e-3 Pa s
m_B	1e-3 kg
m_D	99e-3 kg
e_D	1e-6 m
k_D	2e6 N/m
d_a	1e1 Ns/m
g	9.81 m/s^2

parameter values

m_R	75.4e-3 kg
J_R	6e-8 kg m^2
R_R	2.8e-3 m
k_R	1.26e7 N/m
ξL	18.8 Ns/m
ζL	-2.33e3 Ns/m
m_F	150e-3 kg
ν_F	1e-6 m^2/s
δ	0.9

parameter values

$\bar{m}_B$	5.7e-3	10e-3
$\bar{m}_D$	5.6e-1	9.9e-1
ρ	0.1	
$\bar{k}_D$	11.62	20.39
σ	2.33	4.09
$\bar{d}_a$	57.5e-3	101e-3
$\bar{m}_R$	430e-3	
$\bar{J}_R$	12.47	
$\bar{k}_R$	7.3e1	
$\bar{d}_\xi$	0.108	
$\bar{d}_\zeta$	-13.42	
$\bar{d}_R$	29.9	